Final Exam

Calculus I, Math 161, Fall 2022

Name: ______________________________

Instructor: __________________ **Section No.:** _____

- This exam has 13 questions worth a total of 150 points. Please check that your exam is complete, but otherwise do not look at the exam until the official start.
- You have 120 minutes to complete this exam.
- Fill in your name, section, and instructor above.
- Technology of any kind is prohibited. The use of any notes is prohibited.
- Show your work. Correct work without corresponding work may not receive credit.
- You do **not** need to simplify answers unless specified otherwise. Some specific values of trig functions or e^x or $\ln(x)$ should be known.

Question	Points	Score
1	10	
2	12	
3	20	
4	12	
5	12	
6	12	
7	10	
8	10	
9	10	
10	12	
11	10	
12	8	
13	12	
Total:	150	

1. (10 points) True / False: Indicate True or False with a "T" or "F" (no partial credit).

(a) ___ $\dfrac{1}{x^2+9}$ has vertical asymptotes at $x = -3, 3$.

(b) ___ The limit definition of the derivative is $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$.

(c) ___ If $f(x)$ has a sharp corner at $x = a$, then $f'(a)$ does not exist.

(d) ___ $\lim_{x\to 0^+} \dfrac{\sin(x)}{\ln(x)}$ is of indeterminate form.

(e) ___ $\dfrac{\mathrm{d}}{\mathrm{d}x}\left[\int_2^{x^3} \sqrt{5+t^2+t^5}\,dt\right] = \sqrt{5+x^6+x^{15}}$.

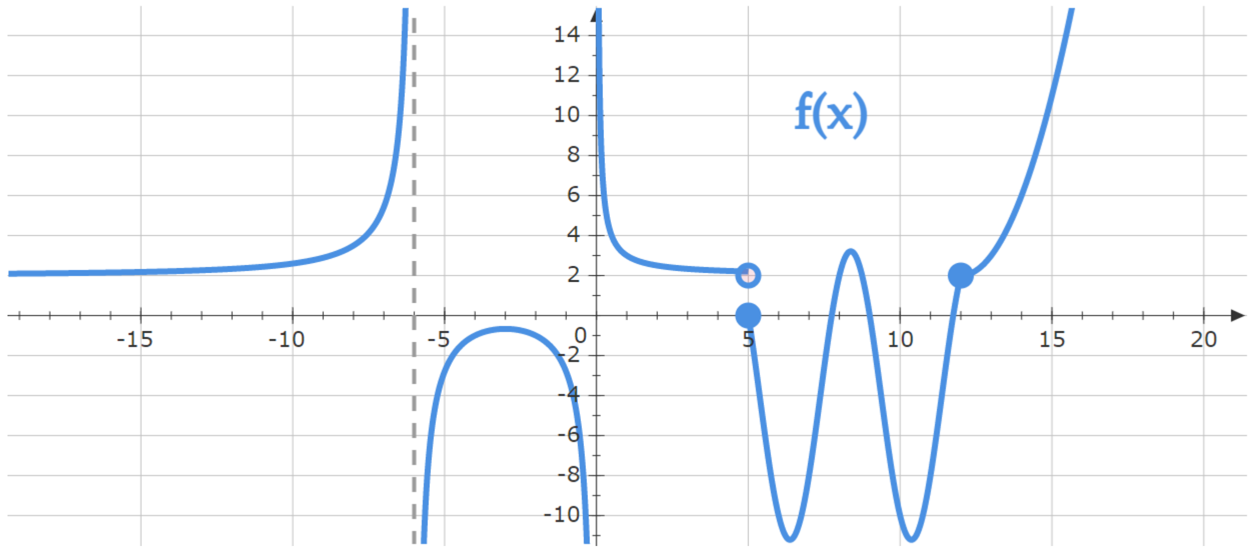
2. (12 points) Evaluate the following limits.

   (a) $\displaystyle\lim_{x \to 2} \frac{x^2 - 4}{x^2 - 5x + 6}$

   (b) $\displaystyle\lim_{x \to -\infty} \frac{2e^x - 5}{3e^x + 7}$

   (c) $\displaystyle\lim_{x \to 1^+} \frac{e^x - e}{\ln x}$

3. (20 points) Use the graph of the function $f(x)$ to answer the following questions:



Reminder: DNE is a valid possible answer.

(a) Find $f(5)$.

(b) Find $f'(12)$.

(c) Find $\lim_{x\to 0} f(x)$.

(d) Is $f(x)$ continuous at $x = 12$?

(e) List all horizontal asymptotes, or write NONE if there are none.

(f) State the x-value where a jump discontinuity occurs.

(g) Find an interval where $f'(x) \geq 0$.

(h) Find an interval where $f''(x) \geq 0$.

(i) Mark all local maxima of $f(x)$ with a $\times$ ("big X") on the graph.

(j) Is there a global maximum?

4. (12 points) Derivatives

   (a) Find y' for $y = \dfrac{\sin(x)}{\cos(x)}$

   (b) Find y' for $y = (x^3 - x)^5$

   (c) Find the **second derivative** y'' for $y = x^7 e^{2x}$

5. (12 points) You own a large sports apparel company, selling Rambler t-shirts. Accounting for all related costs and revenue, the profit in thousands of dollars is given by

$$P(b) = -3 + 8b - b^2,$$

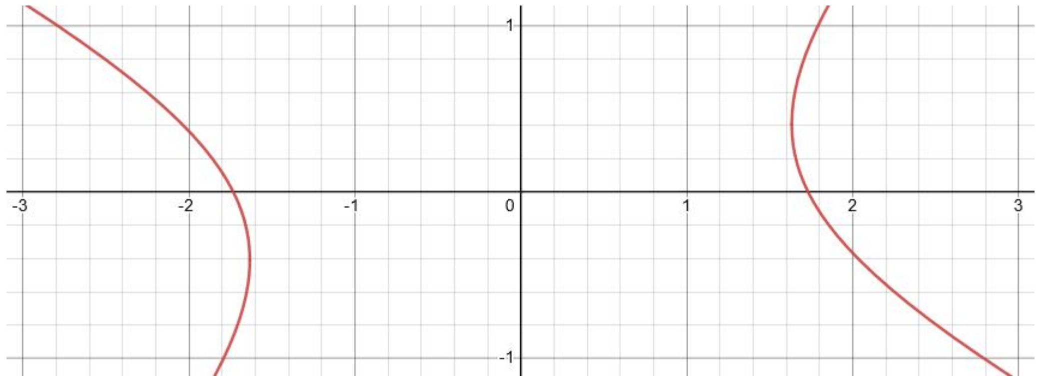
where b represents the number of boxes, measured in thousands, of t-shirts produced.

(a) What is the net change of profit of the company if production increases from $b = 1$ to $b = 3$? Use appropriate units in your answer.

(b) What is the instantaneous rate of change of profit with respect to b when $b = 3$? Use appropriate units in your answer.

(c) Based on your answer in Part (b), you can advise the printshop manager to: increase production, decrease production, or leave production levels unchanged at $b = 3$. What would you advise? Justify your answer.

6. (12 points) The graph of the equation $x^2 + xy = 2y^2 + 3$ is a hyperbola as shown below:



(a) Find y' explicitly in terms of x and y.

(b) Find the equation of the tangent line at the point $(\sqrt{3}, 0)$.

7. (10 points) Find the point on the curve $y = 2\sqrt{x}$ that is closest to the point $(\frac{7}{2}, 0)$. Verify that the extremum found is a minimum by either the first or second derivative test.

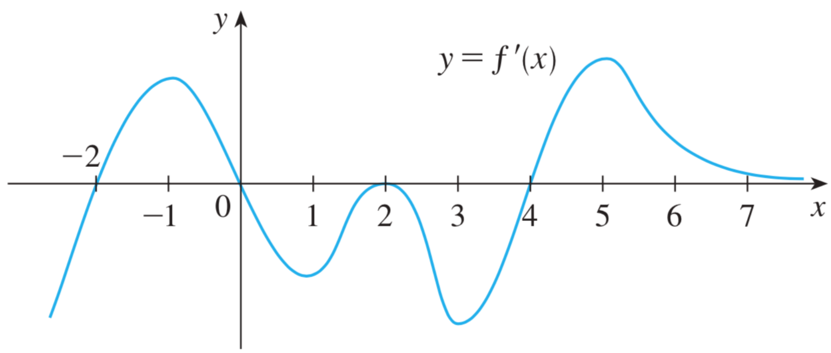
8. (10 points) The volume of a cube is increasing at a rate of $10\,\text{cm}^3/\text{min}$. How fast is the surface area increasing when the length of an edge is $30\,\text{cm}$?
[Recall if L is the length of an edge of the cube, the volume is $V = L^3$ and the surface area is $A = 6L^2$]

9. (10 points) Consider the function
$$f(x) = \ln x.$$

(a) Find the linearization $L(x)$ of $f(x)$ at $x = 3$.

(b) Find the third-order Taylor polynomial centered at $c = 2$ for $f(x)$.

10. (12 points) Use the graph of $f'(x)$ below to answer questions about the **original** function $f(x)$:



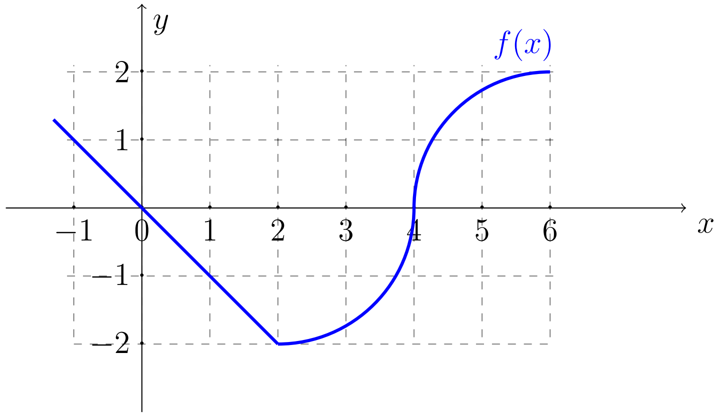
(a) Identify all critical numbers (the x-values are sufficient).

(b) Classify each critical number as corresponding to a local minimum, local maximum, or neither.

(c) On what intervals is $f(x)$ increasing? *Leave your answer as open interval(s). For instance,* $(1, 2)$ *means* $1 < x < 2$.

(d) For what values of x does $f(x)$ have an inflection point?

11. (10 points) $f(x)$ is defined by the graph below:



(a) Consider $A = \displaystyle\int_1^b f(x)\,dx$ for $1 < b \leq 6$. What value of b **minimizes** A? Justify your response.

(b) Consider $\displaystyle\int_4^6 f(x)\,dx$. Which of the following two approximations of this definite integral is larger, a left-hand Riemann sum or a right-hand Riemann sum? (Assume four equal sub-intervals for each.) Justify your response.

12. (8 points) Evaluate the following indefinite integrals:

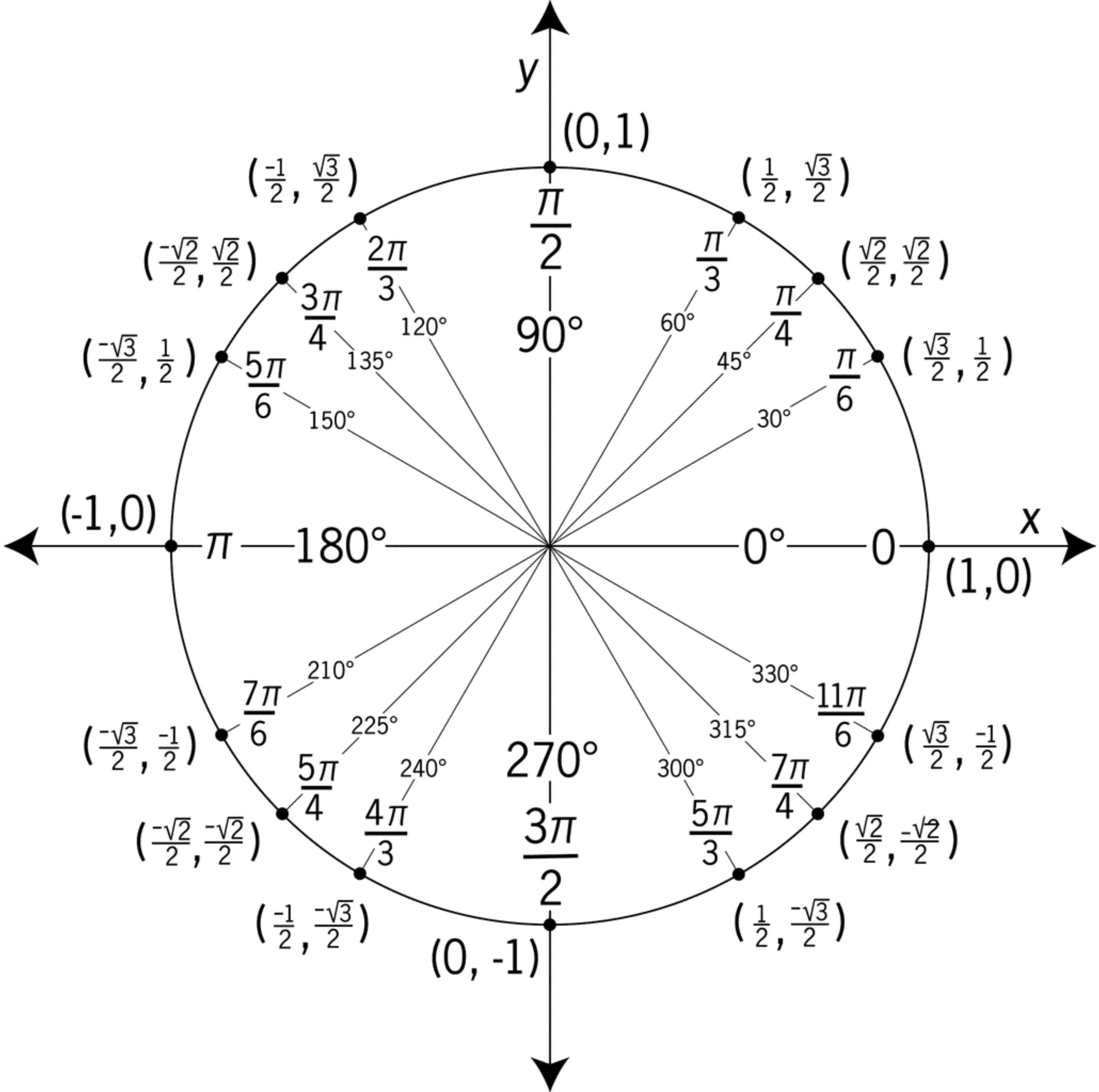
(a) $\displaystyle\int \frac{5x^3 + 3x^2 + 2x + 1}{x^2}\, dx$

(b) $\displaystyle\int \frac{e^{3x}}{1 + e^{3x}}\, dx$

13. (12 points) Evaluate the following definite integrals:

(a) $\displaystyle\int_{\pi/6}^{\pi/2} 1 + \cos(x)\, dx$

(b) $\displaystyle\int_{-7}^{0} \sqrt{49 - x^2}\, dx$ using geometry


y
x
(0,1)
(1,0)
(-1,0)
(0, -1)
0°
90°
180°
270°
30°
45°
60°
120°
135°
150°
210°
225°
240°
300°
315°
330°
0
$\frac{\pi}{6}$
$\frac{\pi}{4}$
$\frac{\pi}{3}$
$\frac{\pi}{2}$
$\frac{2\pi}{3}$
$\frac{3\pi}{4}$
$\frac{5\pi}{6}$
π
$\frac{7\pi}{6}$
$\frac{5\pi}{4}$
$\frac{4\pi}{3}$
$\frac{3\pi}{2}$
$\frac{5\pi}{3}$
$\frac{7\pi}{4}$
$\frac{11\pi}{6}$
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